University of California, Berkeley
Physics 110B Spring 1997 (*Strovink*)

# FINAL EXAMINATION

**Directions.** Do all seven problems, which have unequal weight. This is a closed-book closed-note exam except for two $8\frac{1}{2} \times 11$ inch sheets containing any information you wish on both sides. Calculators are not needed. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Cross out rather than erase any work that you wish the grader to ignore. *You must justify what you do or say.* Express your answer in terms of the quantities specified in the problem. Box or circle your answer. Remember that when you are asked for the value of a vector quantity, you must supply both the magnitude and direction.

**1**. (20 points)
Write down the (real) electric and magnetic fields for a monochromatic plane wave in vacuum of amplitude $E_0$, angular frequency $\omega$, and phase angle 0 (relative to a cosine). Do it for two cases: the wave is

**(a)** (10 points) traveling in the negative-$x$ direction and polarized in the $z$ direction;

**(b)** (10 points) traveling in the direction from the origin to the point (1,1,0), with polarization perpendicular to the $x$ axis.

**2**. (10 points)
Consider a spherical pulsating bubble with constant total charge $Q$ uniformly distributed on the surface, and with time-dependent radius $r(t) = a(1 + \epsilon \cos \omega t)$, where $a$, $\epsilon$, and $\omega$ are constants. Find the total power $P$ that is radiated.

**3**. (35 points)
A particle with charge $e$ moves with speed $\beta c$ around a circle of radius $b$ centered at the origin. The circle is in the plane $z = 0$. The motion is ultrarelativistic, *i.e.* $(1 - \beta^2)^{-1/2} \gg 1$.

Liénard's equation for the Poynting vector $\mathbf{S}_\mathrm{a}$ arising from acceleration of a point particle is

$$\mathbf{S}_\mathrm{a} = (\frac{e}{4\pi\epsilon_0})^2 \frac{\epsilon_0}{c} \Big\{ \frac{\hat{\mathbf{R}}}{R^2} \big[ \frac{\hat{\mathbf{R}} \times [(\hat{\mathbf{R}} - \vec{\beta}) \times \dot{\vec{\beta}}]}{(1 - \hat{\mathbf{R}} \cdot \vec{\beta})^3} \big]^2 \Big\}_\mathrm{ret} .$$

Here $\vec{\beta} c$ is the particle's velocity, $\dot{\vec{\beta}} c$ is its acceleration, $\mathbf{r}$ is a vector from the origin to the observer, $\mathbf{r}'$ is a vector from the origin to the particle, $\mathbf{R} = \mathbf{r} - \mathbf{r}'$, and the subscript "ret" means that quantities are to be evaluated at time $t - R/c$.

**(a)** (20 points) Calculate the radiated power per unit area observed at $(0, 0, z)$, where $z \gg b$.

**(b)** (15 points) Is $\hat{\mathbf{z}}$ a direction in which the power radiated per unit solid angle is near the maximum for this motion? Explain.

**4**. (35 points)
Write down the Fraunhofer diffraction pattern $I(\theta)/I(\theta = 0)$ for monochromatic light of wavelength $\lambda$ normally incident on a system of four thin slits. Two slits are at $y = (a \pm b)/2$, and two are at $y = -(a \pm b)/2$.

**5**. (35 points)
A circularly polarized plane wave of wavelength $\lambda$ is normally incident on a double thin slit (separation $d$). In front of the top slit is placed a quarter wave plate. Obtain the Fraunhofer diffraction pattern $I(\theta)/I(\theta = 0)$. Take the optical thickness of the plate to be such that the irradiance is largest at $\theta = 0$.

**6**. (30 points)
Two perfect parallel mirrors enclose a sandwich consisting of two layers: a dielectric of (real constant) refractive index $n$ between $0 < x < L$, and a region of vacuum between $L < x < (n + 1)L$ . A plane standing EM wave (the sum of two traveling waves with opposite directions of propagation) propagates along the mirrors' normal. Calculate the wave's lowest possible angular frequency.

**7.** (35 points)
A plane wave is normally incident on an opaque screen in the plane $z = 0$. The screen blocks the semi-infinite region $x < 0$. It also has a semicircular protrusion of radius $R$, centered at $x = y = 0$. Thus the screen also blocks the circular region $\sqrt{x^2 + y^2} < R$.

An observer is stationed at $(0, 0, R^2/\lambda)$, where $\lambda$ is the wavelength. Calculate the ratio

$$I_{\text{screen}}/I_{\text{no screen}}$$

of irradiances seen by the observer with and without the screen in place.